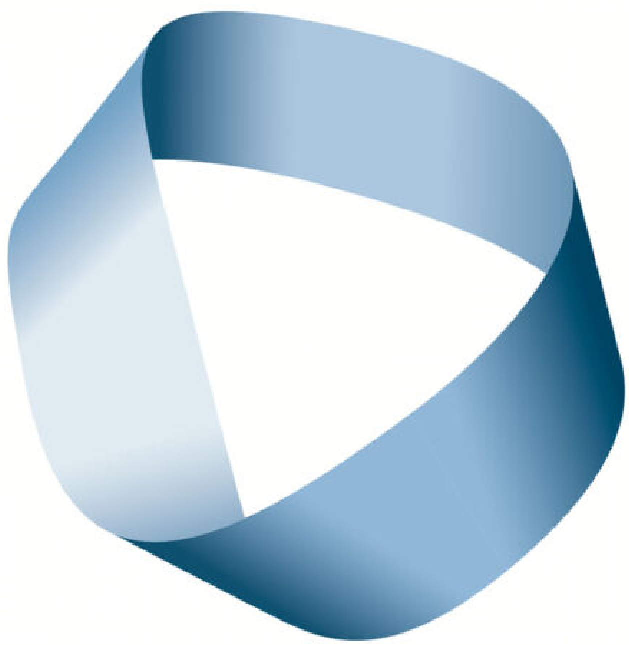
INFOMAT

OKTOBER 2024



Hva er det maksimale forholdet mellom båndbredde og omkrets for et Möbiusbånd?

INFOMAT kommer ut med 11 nummer i året og gis ut av Norsk Matematisk Forening. Deadline for neste utgave er alltid den 15. i neste måned. Stoff til INFOMAT sendes til

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Ansvarlig redaktør er Arne B. Sletsjøe, Universitetet i Oslo

Matematisk kalender

2025

Januar:

13.-17. QUANTUM GROUPS, TENSOR CATEGORIES AND QUANTUM FIELD THEORY, Oslo
<https://www.mn.uio.no/math/english/research/groups/operator-algebras/events/conferences/qg-2025/index.html>

Nye doktorgrader

Jonatan Stava ved UiB forsvarte 25. oktober 2024 sin avhandling *The Connection Algebra of Reductive Homogeneous Spaces* for graden PhD.

Veiledere har vært Hans Munthe-Kaas (hoved-) og Erlend Grong (bi-), begge UiB.

Sammendrag:

Kandidaten gir en beskrivelse av algebraiske strukturer kalt koblingsalgebraer som oppstår naturlig på en klasse matematiske rom kalt reduktive homogene rom. Hovedresultatet er en komplett klassifisering av disse algebraene inkludert en beskrivelse av den tilhørende frie algebraen. Dette kan brukes til å utvikle numeriske metoder for å løse differensiallikninger.

Differensiallikninger oppstår naturlig i mange sammenhenger dersom man forsøker å beskrive alt fra himmellegemers bevegelse til økonomisk vekst. Disse likningene kan være svært vanskelig å finne eksakte løsninger til, og man er derfor avhengig av gode numeriske metoder for å finne tilnærmede løsninger. Runge-Kutta metoder er en klasse med slike metoder, men disse metodene er tilpasset flate matematiske rom. I CODYSMA-prosjektet har en gruppe forskere lagt grunnlaget for å tilpasse disse metodene for matematiske rom som ikke trenger å være flate, for eksempel en sfære.

Disse rommene er utstyrt med et matematisk objekt kalt en kobling som lar oss ta den retningsderiverte av vektorfelt. Vi tenker på denne koblingen som et produkt som lar oss multiplisere vektrofelt med hverandre. Basert på geometrien til det matematiske rommet vi jobber på vil vi få en rekke regneregler. Viktige eksempler på slike regneregler som vi kjenner godt er kommutativitet: «ab = ba», og assosiativitet: «a(bc)=(ab)c». I de tilfellene vi undersøker er disse reglene ikke nødvendigvis gjeldende. Regnereglene vi får definerer en type algebra, og vi er interessert i å vite nøyaktig hvilke konsekvenser et sett med regneregler medfører. Dette undersøker vi ved å se på det som kalles den frie algebraen. Dette er en algebra hvor vi passer på at ingen andre regler gjelder enn de vi er interessert i, det vil si at vi jobber et sted hvor for blant annet «ab» ikke er lik «ba». Den frie algebraen kan ses på som en form for variabel kontroll; vi fremstiller en destillert versjon av strukturen vi er interessert i.

Belén García Pascual ved UiB forsvarte 25. oktober 2024 sin avhandling *Mathematical Modelling of Cellular and Evolutionary Processes in Changing Environments* for graden PhD.

Veiledere har vært Iain Johnston, Jan Nordbotten og Konstantinos Giannakis, alle UiB.

Sammendrag:

Organismer tilpasser seg sitt miljø for å ha større sjanse for å overleve. Men, hvordan tilpasser de seg hvis miljøet endrer seg på relativt kort tid? Organismer og deres forhold til miljøet er komplekse, så vi trenger metoder for å utforske disse systemene og som kan kobles til observasjoner fra data. Matematiske modeller hjelper oss til å forstå biologiske systemer, forutse atferder og teste hypoteser.

I denne avhandlingen studerer vi evolusjonære biologiske systemer med et sentralt fokus i energi- og utviklingsprosesser. I cellen finnes mitokondrier og plastider som er organeller ansvarlige for å gi energi til å drive alle prosessene nødvendige for liv. Noen arter har mange gener i organellene, mens andre har veldig få og trenger i større grad genene fra cellekjernen. Vår matematiske modell av ordinære differensiallikninger viser at det er bedre å ha gener i organellene hvis miljøet kraftig og periodisk endres, og bedre å ha gener i cellekjernen hvis miljøet er mer stabilt. Vi forsetter med en evolusjonær beregningsmodell for å studere arvstrategier for organellegener. Modellen viser at arv fra et mellomliggende antall kopier av gener fra både mor og far bidrar til større variasjon av ulike funksjonelle gener i ulike miljøer.

Vi studerer sjøanemoner som et eksempel på hvordan en hel organisme utvikler seg i et endrende miljø. Disse dyrene vokser når de blir matet og krymper når de sulter. Fra flerfasede lineære regresjoner ser vi at kroppsendringene stort sett skjer på samme måte, uavhengig av faktorer som tidligere vekstmønstre.

Et viktig problem er hvordan bakterier tilpasser seg miljøendringer i form av leggemiddeleksponering. Dette fører til leggemiddelresistens som påvirker helsen vår. Markov-overgangsgrafer beskriver mulige rekkefølger av resistens. Vi utvikler en sammenligningsmetode som gir en kurve, og formene på kurven rapporterer forskjellene mellom grafene. Dette hjelper oss til å se forskjellene mellom legemiddelresistens i ulike deler av verden.

Kunngjøringer

Dear colleagues

It has been a longstanding tradition that the organizers of the ICM offer support in order that mathematicians from developing countries can participate at the ICM. For ICM 2026 in Philadelphia, USA, the organizers have announced the following grant program:

ICM 2026 Travel Support Program

With generous funding from the Simons Foundation and in collaboration with the IMU and the IMU´s Commission for Developing Countries (CDC), the American Mathematical Society (AMS) is offering travel support for ICM 2026 participants that will provide partial support for mathematicians from eligible developing countries to attend the ICM.

The application deadline is **20 November 2024 (11:59 pm EST).** Applicants will be notified of their application status by 31 March 2025. Further information can be found on the ICM 2026 website.

Arrangementer

QUANTUM GROUPS, TENSOR CATEGORIES AND QUANTUM FIELD THEORY, Oslo 13.-17. januar 2025

Quantum field theory has been for a long time a big driving force in many areas of mathematics. In particular, it stimulated a fruitful study of mathematical structures that can represent many-body quantum mechanical systems, such as quantized universal enveloping algebras, inclusions of operator algebras and tensor categorical structures governing them, which are also interesting for their own sake. This workshop aims to bring together international experts on these topics, stimulate further exchanges of research ideas, and give an opportunity for young members of our research community to be exposed to the rapidly evolving research at the top level.

David Jaklitsch, Sergey Neshveyev and Makoto Yamashita

Nyheter

NYTT STORT PRIMTALL

The largest known prime number is $2^{136,279,841} - 1$, a number which has 41,024,320 digits when written in base 10. It was found on October 12, 2024 by a computer volunteered by Luke Durant to the Great Internet Mersenne Prime Search (GIMPS).

OM MÖBIUSBÅND

(Artikkelen er sakset fra en artikkel i Scientific American, skrevet av Rachel Crowell)

The 1858 discovery of Möbius bands is credited to two German mathematicians – August Ferdinand Möbius and Johann Benedict Listing – though evidence suggests that mathematical giant Carl Friedrich Gauss was also aware of the shapes at this time, says Moira Chas, a mathematician at Stony Brook University. Regardless of who first thought about them, until recently, researchers were stumped by one seemingly easy question about Möbius bands: What is the shortest strip of paper needed to make one? Specifically, this problem was unsolved for smooth Möbius strips that are “embedded” instead of “immersed,” meaning they “don’t interpenetrate themselves,” or self-intersect, says Richard Evan Schwartz, a mathematician at Brown University. Imagine that “the Möbius strip was actually a hologram, a kind of ghostly graphical projection into three-dimensional space,” Schwartz says. For an immersed Möbius band, “several sheets of the thing could overlap with each other, sort of like a ghost walking through a wall,” but for an embedded band, “there are no overlaps like this.”

In 1977 mathematicians Charles Sidney Weaver and Benjamin Rigler Halpern posed this question about the minimum size and noted that “their problem becomes easy if you allow the Möbius band you are making to have self-intersections,” says Dmitry Fuchs, a mathematician at the University of California, Davis. The remaining question, he adds, “was to determine, informally speaking, how much room you need to avoid self-intersections.” Halpern and Weaver proposed a minimum size, but they couldn’t prove this idea, called the Halpern-Weaver conjecture.

Schwartz first learned about the problem about four years ago, when Sergei Tabachnikov, a mathematician at Pennsylvania State University, mentioned it to him, and Schwartz read a chapter on the subject in a book Tabachnikov and Fuchs had written. “I read the chapter, and I was hooked,” he says. Now his interest has paid off with a solution to the problem at last. In a preprint paper posted on arXiv.org on August 24, 2023, Schwartz proved the Halpern-Weaver conjecture. He showed that embedded Möbius strips made out of paper can only be constructed with an aspect ratio greater than $\sqrt{3}$, which is about 1.73.

In Schwartz’s proof, “Rich managed to dissect the problem into manageable pieces, each of which essentially necessitated only basic geometry to be solved,” says Max Wardetzky, a mathematician at the University of Göttingen in Germany. “This approach to proofs embodies one of the purest forms of elegance and beauty.”

Before arriving at the successful strategy, however, Schwartz tried other tactics on and off again over a few years. He recently decided to revisit the problem because of a nagging sensation that the approach he had used in a 2021 paper should have worked.

In a way, his gut feeling was correct. When he resumed investigating the problem, he noticed a mistake in a lemma, involving a “T-pattern” in his previous paper. By correcting the error, Schwartz quickly and easily proved the Halpern-Weaver conjecture. If not for that mistake, “I would have solved this thing three years ago!” Schwartz says.

In Schwartz’s solution to the Halpern-Weaver conjecture, the T-pattern lemma is a critical component. The lemma begins with one basic idea: “Möbius bands, they have these straight lines on them. They’re called ruled surfaces,” he says. You can imagine drawing these straight lines so that they cut across the Möbius band and hit the boundary at either end.

In his earlier work, Schwartz identified two straight lines that are perpendicular to each other and also in the same plane, forming a T-pattern on every Möbius strip. “It is not at all obvious that these things exist,” Schwartz says. Showing that they do was the first part of proving the lemma, however.

The next step was to set up and solve an optimization problem that entailed slicing open a Möbius band at an angle (rather than perpendicular to the boundary) along a line segment that stretched across the width of the band and considering the resulting shape. For this step, in Schwartz’s 2021 paper, he incorrectly concluded that this shape was a parallelogram. It’s actually a trapezoid.

Last summer, Schwartz decided to try a different tactic. He started experimenting with squishing paper Möbius bands flat. He thought, “Maybe if I can show that you can press them into the plane, I can simplify it to an easier problem where you’re just thinking of planar objects.”

During those experiments, Schwartz cut open a Möbius band and realized, “Oh, my God, it’s not the parallelogram. It’s a trapezoid.” Discovering his mistake, Schwartz was first annoyed (“I hate making mistakes,” he says) but then driven to use the new information to rerun other calculations. “The corrected calculation gave me the number that was the conjecture,” he says. “I was gobsmacked.... I spent, like, the next three days hardly sleeping, just writing this thing up.”

Finally, the 50-year-old question was answered. “It takes courage to try to solve a problem that remained open for a long time,” Tabachnikov says. “It is characteristic of Richard Schwartz’s approach to mathematics: He likes attacking problems that are relatively easy to state and that are known to be hard. And typically he sees new aspects of these problems that the previous researchers didn’t notice.”